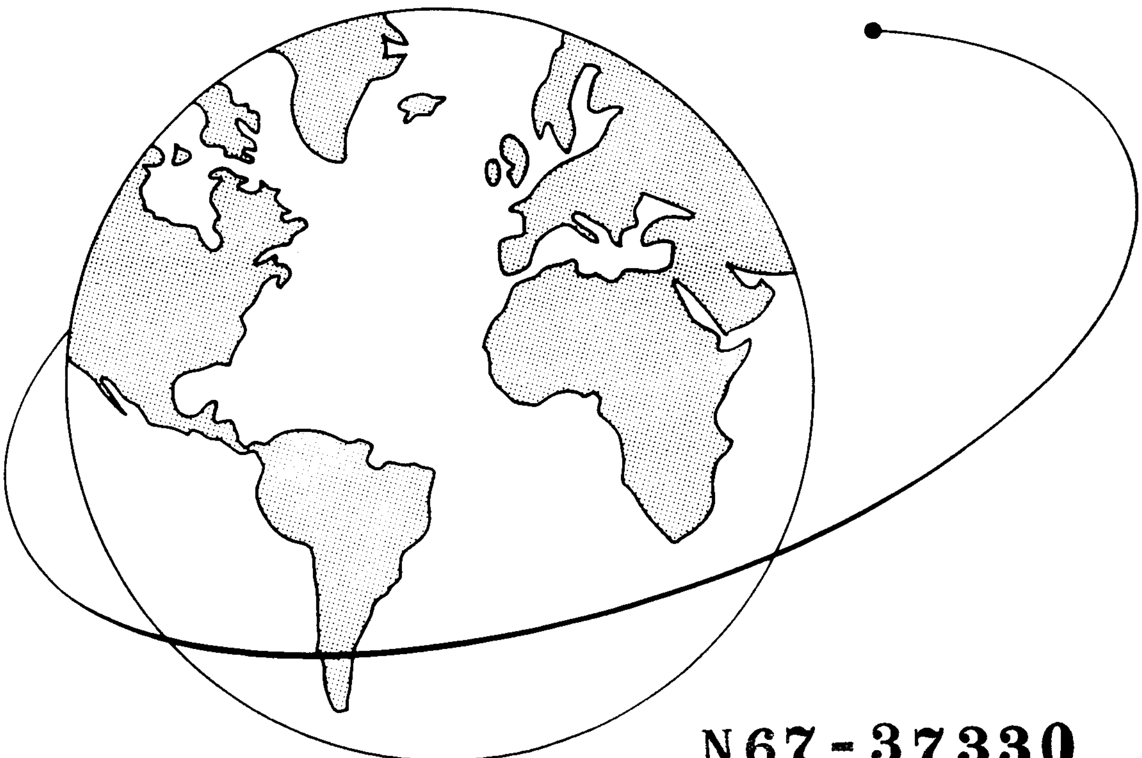
SATELLITE ALTIMETRY AND ORBIT DETERMINATION

C. A. LUNDQUIST



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ABSTRACT

The technological problems associated with altimetry from a satellite are the subject of wide investigation in the United States. Therefore, it is reasonable to assume that accurate altitudes measured from satellites will eventually be available, and it is prudent to ask now what are the interfaces between altitude measurements and satellite-orbit-determination practices. From one point of view, accurate altitude data may generate accuracy requirements that must be met by orbit-determination procedures. From another point of view, the altitudes themselves may be used as tracking data in orbit determination. If the altitude of a satellite above the ocean surface is obtained, this may be viewed as a measured relationship between a point on an equipotential surface of the geopotential and a satellite position determined by the equations of motion derived from the geopotential. These various interfaces can be explored in the context of the procedures used at the Smithsonian Astrophysical Observatory (SAO) for orbit determination and geophysical research.

RÉSUMÉ

Les problèmes technologiques associés à l'altimétrie par satellite sont l'objet de vastes investigations aux Etats-Unis. Il est par conséquent raisonnable d'admettre que des altitudes précises mesurées à partir des satellites seront finalement disponibles et il est prudent de demander maintenant quels sont les interfaces entre les mesures d'altitude et les pratiques de détermination d'orbite des satellites. D'un premier point de vue, des données précises d'altitude peuvent créer un besoin d'exactitude qui doit être satisfait par des procédés de détermination de l'orbite. D'un autre point de vue, les altitudes elles-mêmes peuvent être utilisées comme des données de poursuite dans la détermination de l'orbite. Si l'on obtient l'altitude d'un satellite au-dessus de la surface de l'océan, ceci peut être considéré comme une relation mesurée entre un point sur la surface équipotentielle du géopotentiel et une position du satellite déterminée par les équations de mouvement déduites du géopotentiel. Ces différents interfaces peuvent être explorés dans le contexte des procédés employés à l'Observatoire d'Astrophysique du Smithsonian pour la détermination des orbites et la recherche géophysique.

Конспект

Технологические проблемы, связанные с альтиметрией, произведенной из спутника, являются объектом широкого исследования в Соединенных Штатах. Поэтому разумно предположить что точные высоты, измеренные из спутников будут со временем доступны и является благоразумным сейчас поставить вопрос о том какие существуют грани между методами измерений высоты и определения орбиты. С одной стороны, точные данные высоты могут вызывать требования точности,которая должна быть осуществлена методами определения орбиты. С другой стороны, сами высоты могут употребляться как данные наблюдений для определения орбиты. Если высота спутника над поверхностью океана получена она может рассматриваться как измеренное взаимоотношение между точкой на эквипотенциальной поверхности геопотенциала и положением спутника, определенным уравнениями движения, выведенными из геопотенциала. Эти различные грани могут быть исследованы в контексте образа действий употребляемых в Смитсониан Астрофизической Обсерватории для определения орбиты и геофизического исследования.

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Charles A. Lundquist

1. INTRODUCTION AND HISTORY

Within the brief span of the space age, satellite-borne altimeters are an old idea. Interest in instrumentation to measure the altitude of a spacecraft has at least two principal motivations. One branch of activity has roots in the proposal that on-board altimeters can provide useful information to a vehicle-guidance system. A second branch stems from a desire to measure the geometrical shape of the ocean surface and its variations.

For applications near the earth, most altimeter-based guidance schemes would use the ocean surface as a reference from which to measure the space vehicle position (e. g., Godbey and Roeder, 1962; Speer and Kurtz, 1963). A similar philosophy prevails in suggestions to use an altimeter for diagnostic tracking during vehicle-development tests or critical orbital operations (e. g., Hoffman and Olthoff, 1963). For particular guidance or tracking accuracy requirements, this point of view implies that the ocean-surface geometry must be known with corresponding accuracy. Typically, space-vehicle engineers expect oceanography to provide the necessary description of sea level.

From their own point of view, various oceanographers (and geophysicists) are interested in the shape of the ocean to differing degrees of detail (Frey, Harrington, and von Arx, 1965). In the open ocean, they believe that the ocean has a static, equipotential surface to within a meter or so. Thus a

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representation of the surface geometry to this accuracy reflects structural detail within the solid earth. For example, relative to a spheroid, sea level has about a 15-m dip in a degree of latitude across the Puerto Rico trench (von Arx, 1966). However, at decimeter accuracy, sea level varies owing to many dynamical processes — tides, cyclones, currents, etc. (Woollard, 1966). Oceanographers studying these dynamical effects assert that if instrumentation does not permit measurement of relative elevation to within 50 cm, the information is of essentially no oceanographic use (Stewart, 1965).

If satellite altimeters can approach an accuracy of 1 m, scientists concerned with deducing information about the solid earth beneath the sea become very interested; if altimetry eventually reaches decimeter accuracy, the dynamical oceanographers also become excited. In either case, they hope that practioners of celestial mechanics and satellite tracking will provide absolute satellite positions of sufficient accuracy so that the positions can be used as a reference from which to deduce sea level.

Such hopes by oceanographers on the one hand and reciprocal expectations by mechanicians on the other could carry the beginnings of a chicken-and-egg attitude toward the use of altimetry data: Which comes first, an accurate geoid or accurate orbits? The actual situation is not quite this extreme, fortunately, and several authors point out that geoid and orbit improvements can proceed together (Godbey, 1965; Frey et al., 1965; Rouse, Waite, and Walters, 1966; Lundquist, 1967). The purpose of this paper is to outline one way in which this process of mutual improvement could develop naturally. The outlined process follows the established practice in satellite geodesy.

Before a discussion of the procedures, a few background remarks about altimeter hardware are in order. The systems flown and proposed to date transmit an electromagnetic signal from the spacecraft toward the ocean surface or the solid surface from whence a reflected signal returns to the satellite. The transit time, corrected for atmosphere effects, measures the altitude of the spacecraft above the reflecting surface. The electromagnetic radiation can have radio frequency, light frequency, or some other frequency.

Radar altimeters for spacecraft are mostly an outgrowth of similar aircraft systems. However, the first satellite experience with reflections from the earth was a by-product of the swept-frequency topside sounder carried for ionosphere research on the Canadian Alouette launched on September 29, 1962 (Molozzi, 1964). In addition to returns from the ionosphere, many ionograms contained returns from the earth at frequencies above the critical frequency of the ionosphere (Chia, Doemland, and Moore, 1967; Moore, 1965). An altimeter designed for vehicle tracking flew in Saturn SA-4 in March, 1963 (Hoffman and Olthoff, 1963; Dugan, 1963). Preliminary designs for other systems are documented in more recent papers (Godbey, 1965; Frey et al., 1965; Westinghouse, 1966).

Over the ocean, the accuracy of a radar altimeter is related intimately to the reflecting character of the sea surface with its variable wave structure. Satellite measurement of the sea state – i. e., wave size – is an interesting topic, which has been widely discussed (e. g., Pierson, 1965). Perhaps it is fair to say, in summary, that altitudes over the ocean to an accuracy approaching a meter or so represent a reasonable expectation in future radar systems. But a note of caution is appropriate, because experience with range measurements between ground stations and active satellite transponders indicates that even for this case meter accuracy is difficult to obtain at radio frequencies.

Laser altimeters are a newer concept. Possible laser uses in guidance schemes are touched on briefly in several documents (e.g., Walker, 1965; Wyman, 1965). A spacecraft altimeter has been studied and experiments performed from an aircraft over the ocean (Raytheon Company, 1967).

Although no laser altimeter has flown yet in a spacecraft, a laser transmitter for a communication experiment was developed and carried on the Gemini-7 flight (Radio Corporation of America, 1965; Piland and Penrod, 1966).

Several potential laser systems, including ruby lasers for which much related experience exists, hold promise for achieving meter-accuracy altitudes when cloud cover permits. In a comparison of lasers with radars, this cloud-cover limitation is offset to some degree by the realization that ground-station satellite tracking with lasers is the only technique now routinely producing range data to meter accuracy (Plotkin, 1965; Lehr, 1966).

Altimeter applications to lunar problems are similar in many respects to earth problems, but differ in the important respect that the moon has no ocean to serve as a reference surface. For this latter reason, the lunar situation will not be considered further here other than to say that common hardware may be developed for use near the earth, moon, and other planets.

In summary, satellite altitudes above the ocean surface must be measured to an accuracy better than 10 m if these data are to be valuable. One-m accuracy is a reasonable objective to adopt for a first step (NASA, 1967), although neither a radar nor a laser system of this quality has been demonstrated.

2. UTILIZATION OF ALTITUDE DATA

Satellite geodesy has matured to the state where it is a recognized branch of geodetic science with its own established procedures. The conventional cycle of analysis begins at a tracking station, which measures some quantity depending on satellite position or velocity. Each observation yields an equation of condition relating orbital elements and geodetic parameters. Very many such equations are used to refine orbital elements and geodetic parameters, either simultaneously or cyclically. In these solutions the equations need not all arise from measurement of the same function of satellite position or velocity. Rather, a blend of data from various tracking systems can strengthen the solution.

My point here is that altimeter data can be blended into the same procedures with no essential change in philosophy or computer programs. The latter is particularly important since the computer programs in use by various investigators are all rather substantial, and any alternate program to use accurate altitudes will have to have comparable complexity. Another consequence is that satellite orbits and the geoid can be obtained simultaneously from altitude data, in the same way that present orbits and geopotential representations are derived together from tracking data. The accuracies are compatible also. For example, programs in advanced stages of development at the Smithsonian Astrophysical Observatory (SAO) for using laser range data are written to maintain a precision of 0.5 m.

In the paragraphs below, I outline the formulation for blending altitudes into the procedures practiced at SAO, following the development given by E. M. Gaposchkin (1966) for range observations; where possible, the notation is from the same source.

The suggested approach to altimetry adopts the assumption that sea level, averaged over wave structure, is an equipotential surface to an accuracy of approximately a meter. This is also about the accuracy that can reasonably be expected from future altimeter hardware. Thus, dynamical effects in the ocean are neglected for the present. The equipotential surface corresponding to sea level, i. e., the geoid, is represented by

$$U = \frac{GM}{r} \left\{ 1 + \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n C_{n0} P_{n0}(\sin \phi) \right.$$

$$\left. + \sum_{n=2}^{\infty} \sum_{m=1}^{n} \left(\frac{a}{r}\right)^n \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda\right) P_{nm}(\sin \phi) \right\}$$

$$+ \frac{\omega^2 r^2}{2} \cos^2 \phi = C_0, \text{ a constant}$$

where

GM	= gravitational constant for the earth,
C_{nm}, S_{nm}	= harmonic coefficients for the geopotential,
a	= reference equitorial radius of earth,
ω	= rotational rate of earth,
r	= geocentric radius to satellite,
ϕ	= geocentric latitude,
λ	= longitude.

In the coordinate system used for the orbit theory (essentially an inertial system), Figure 1 illustrates definitions of further notation. Note particularly that the earth rotates in this system, but the geocentric vector $\vec{s}$ to the sea surface is expressed in a space-fixed system. The corresponding vector in earth-fixed coordinates is $\vec{S}$, which is related to $\vec{s}$ by transformations $\mathcal{R}_s(\theta)\mathcal{R}(x, y)$; these specify the rotation of the earth and polar motion, respectively (Gaposchkin, 1966).

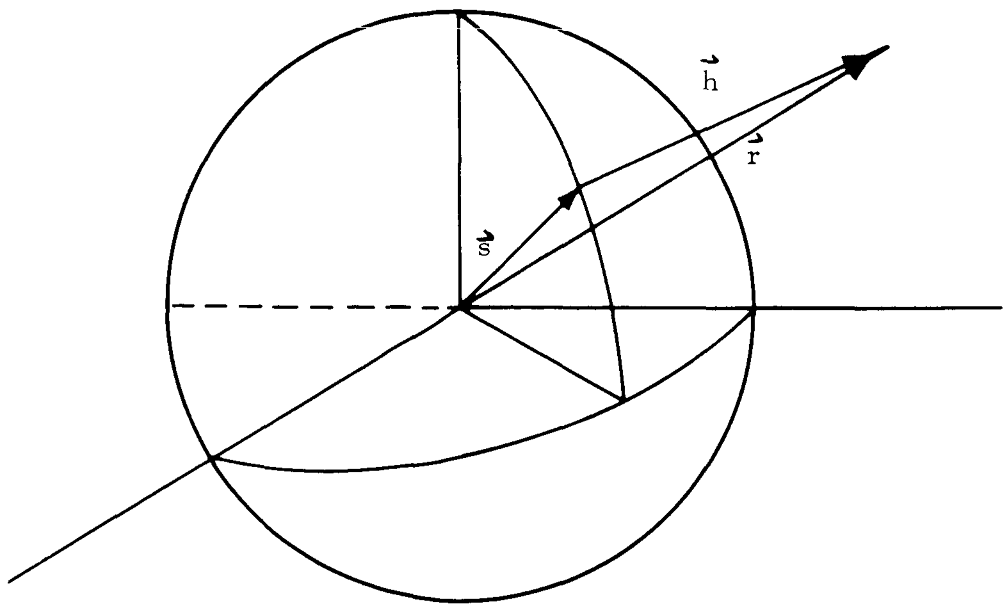

Figure 1. Relation between $\vec{r}$, $\vec{s}$, and $\vec{h}$.

where

$\vec{r}$ = satellite position,

$\vec{s}$ = position on sea surface,

$\vec{h}$ = altitude,

and

$$\vec{h}(t) = \vec{r}(t) - \vec{s}(t),$$
$$= \vec{r} - \mathcal{R}_3(\theta)\mathcal{R}(x, y)\vec{S}.$$

Further characterization of the altimeter system is necessary before proceeding. One alternative is a beam broad enough to include the point on the sea closest to the satellite. The time of the first return as sensed on the satellite gives the distance to the closest point. This system is characterized mathematically by the condition that $\vec{h}$ is normal to the geoid at $\vec{s}$. The gradient of U, rotated into the correct position at the time of observation, gives the family of normals to U, one of which contains the satellite. The resulting equations can be solved for the vector $\vec{s}$ or $\vec{S}$ as a function of $\vec{r}(t)$ and the parameters C_i (= C_{nm}, S_{nm}, C_o) in U.

A second alternative for the altimeter system assumes a satellite with a stabilized gravity gradient, and a narrow-beam system aligned with the vehicle axis. This system is characterized by the condition that $\vec{h}$ has the direction of the gravity gradient at $\vec{r}$. Again the resulting equations can be solved for $\vec{s}$.

A third alternative would use an active attitude or pointing control on the satellite to characterize the direction of $\vec{h}$.

In all the alternate-system characterizations, the information is sufficient to determine

$$\vec{s} = \vec{s}(\vec{r}, C_i, t) \quad .$$

Since the series for U must be very long, the solution for $\vec{s}$ presumably will be performed by a computer subroutine. For the rest of the discussion here it is sufficient to know that $\vec{s}$ and its derivatives can be computed without particular trouble once an altimeter system has been selected.

For ground-station tracking (following Gaposchkin, 1966) an equation of condition is expressed as

$$A(\vec{\rho}' - \vec{\rho}) = A \frac{\partial \vec{\rho}}{\partial p_i} \Delta p_i \quad ,$$

where

$\vec{\rho}$ = calculated range vector to satellite,

A is an operator such that

$A\vec{\rho}'$ = observed positional quantity,

and

p_i = parameter to be refined.

For satellite altimetry, the corresponding equation is

$$B(\vec{h}' - \vec{h}) = B\,\frac{\partial \vec{h}}{\partial p_i}\,\Delta p_i \quad ,$$

where $B\vec{h}'$ = observed altitude, h'. The operator B in this case may be written, in terms of computed quantities, as

$$B = \frac{\vec{h}}{h}\cdot \quad ,$$

so that

$$h' - h \equiv \Delta h = \frac{\vec{h}}{h}\cdot\left(\frac{\partial \vec{h}}{\partial p_i}\right)\Delta p_i \quad .$$

If E_i represents the conventional orbital elements, then the usual orbit theory used in satellite geodesy gives (Gaposchkin, 1966)

$$\vec{r} = \vec{r}(E_i, C_i, t) \quad .$$

Expanding the equations of condition gives, for ground-station tracking from position $\vec{R}$,

$$A\left[\vec{\rho}' - \vec{r}(E_i, C_i, t) + \mathcal{R}_3(\theta)\mathcal{R}(x, y)\vec{R}\right]$$

$$= A\left[\frac{\partial \vec{r}}{\partial E_i}\,\Delta E_i + \frac{\partial \vec{r}}{\partial C_i}\,\Delta C_i - \mathcal{R}_3(\theta)\mathcal{R}(x, y)\Delta\vec{R}\right] \quad ,$$

and for altimetry

$$B\left[\vec{h}' - \vec{r}(E_i, C_i, t) + \mathcal{R}_3(\theta)\mathcal{R}(x, y)\vec{S}\right]$$

$$= B\left[\left(\frac{\partial \vec{r}}{\partial E_i} - \mathcal{R}_3\mathcal{R}\frac{\partial \vec{S}}{\partial r_j}\frac{\partial r_j}{\partial E_i}\right)\Delta E_i\right.$$

$$\left.+\left(\frac{\partial \vec{r}}{\partial C_i} - \mathcal{R}_3\mathcal{R}\frac{\partial \vec{S}}{\partial C_i} - \mathcal{R}_3\mathcal{R}\frac{\partial \vec{S}}{\partial r_j}\frac{\partial r_j}{\partial C_i}\right)\Delta C_i\right] .$$

The equations of condition arising from tracking and altimetry have exactly similar forms, except that station positions are not involved in the latter. All the expressions in the altimetry equation will already have been programmed for the tracking case, except the expression involving $\vec{s}$ and its derivatives. These depend upon characterization of a particular altimeter system.

From the similarity of the equations it would seem quite easy to blend altimeter observations with the other information from tracking, but some details of course need to be examined further. For example, the geoid will surely depend sensibly upon very many more harmonic coefficients than does the orbit. This is a strength of altimetry, since it will allow a more detailed representation of the geopotential. It may also create problems, if vast numbers of measured altitudes are required to obtain a reasonable solution for very many harmonic coefficients. The lack of data over continents may raise other troubles, since uniform data coverage is probably quite necessary to a uniform representation of the geopotential. In the case of continents, surface-gravity data can perhaps augment altitude data from the oceans. Kaula (1966) has already demonstrated that gravity data can be combined with satellite determinations of the geopotential. Gravity data from oceans can provide an interesting check on the results from altimetry. Fortunately, it will be quite easy to explore many of these questions by the use of the existing programs with slight modifications to simulate altimetry information.

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BIOGRAPHICAL NOTE

CHARLES A. LUNDQUIST joined the Smithsonian Astrophysical Observatory as Assistant Director for Science in 1962. In this position, he is responsible for organizing and coordinating current research projects, as well as seeking new direction for future research.

From 1956 to 1960 he was Chief of the Physics and Astrophysics Section, Research Projects Laboratory, Army Ballistic Missile Agency; and from 1960 to 1962 he held concurrent positions as Director of the Supporting Research Office, and Chief of the Physics and Astrophysics Branch of the Research Projects Division at the Marshall Space Flight Center.

Dr. Lundquist received his undergraduate degree from South Dakota State College in 1949, and his doctorate in 1954 from the University of Kansas.